and the thermodynamic equilibrium and stability criteria in terms of this energy are evident.

### 2.3.  The Local Energy

It is the intent of this development to use the energy

$$E = U - \int \vec{H}_e \cdot \vec{M} \, dV \qquad (2.7)$$

along with the thermodynamic equilibrium and stability criteria to predict equilibrium states for a rigid anisotropic ferromagnet.  To do this, an explicit expression for $U$ in terms of pertinent internal coordinates must be obtained.  $U$ can be written

$$U = E_D + \int \varepsilon_{LOC} \, dV \qquad (2.8)$$

where $E_D$ is either of the previously derived self energy expressions in Equation (2.4) or Equation (2.5).  The remaining energy depends only on local conditions and can be written as the volume integral of an energy density. It is this term that is now of concern.

The method used to obtain this local energy expression is to rely on physics and microscopic models of the magnetic material to guide in the selection of mathematical forms and independent variables for the local energy. Phenomenological methods such as convenient expansions and symmetry requirements are then used to deduce precise forms for the energy expression.[5]

The dominant local energies in a ferromagnet have been classified as the exchange energy, the magnetocrystalline anisotropy energy, the magnetoelastic energy, and the elastic energy.[4]  Each will be considered in order.

The exchange energy has a purely quantum mechanical origin.  It can be traced to the requirement of antisymmetry of the electronic state of the

magnetic ions under interchange of any two electrons.[18]  In considering the interaction of any two magnetic ions, the antisymmetry requirement produces a splitting of energy levels making parallel and antiparallel spin alignment energetically separate.  In a magnetic material at normal temperatures, only the lower lying energy states will be abundantly populated so the complete Hamiltonian may be replaced by an "effective spin Hamiltonian" which has as its energy eigenvalues these several low lying states.[19]  Between any two magnetic ions, this effective spin Hamiltonian can be written as

$$\mathcal{H}_s = -2J_{ij}\vec{S}_i \cdot \vec{S}_j.$$

$J_{ij}$ is the exchange integral which determines the splitting of the low lying states.[20]  If $J_{ij}$ is a positive quantity, parallel spin is a lower energy state and ferromagnetism results.  For $J_{ij}$ negative, antiparallel spin is a lower energy state and antiferromagnetism results.  The Hamiltonian for the entire crystal is

$$\mathcal{H}_s = -2 \sum_{ij} J_{ij}\vec{S}_i \cdot \vec{S}_j.$$

This exchange energy gives rise to a very strong but short range interaction causing a cooperative alignment of magnetic dipoles and hence a spontaneous macroscopic magnetization in the material.  The magnitude of this magnetization has been found, with few exceptions, to be isotropic[21] and to depend mainly on the temperature[22] and to some extent on the pressure in the medium.[23] The small pressure dependence is considered in Section 4.5  To make an adequate selection of thermodynamic variables upon which the macroscopic expression for the exchange energy will depend, one can look to the result of a simple model.  The following model will suggest that the gradients of the components of the magnetization or, alternatively, the gradients of the